Technical Comments

G 80-067 refers to

Comment on "Dynamics of a Flexible Body in Orbit"

G 80-013 (P)

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IN Ref. 1, it is concluded that the flexural vibrations of a slender straight beam stabilized by gravity gradient may be unstably coupled with rigid-body pitch motion if the ratio of the flexural vibrational frequency to the orbital angular velocity, ω_n/ω_c, is near unity.

This conclusion is incorrect in at least two respects. In the first place, the frequency of the lowest bending mode is considerably higher than the orbiting angular velocity. In the second place, $\omega_n/\omega_c \simeq 1$ is not the condition for instability.

A lower limit for flexural vibration frequencies can be established as follows. In the limit, as the beam becomes more slender, its equations of motion in the pitch direction approach that of a tensioned string. If the mass distribution is uniform, then in the limit of slenderness the vibration frequencies approach

$$\omega_n = \omega_c \sqrt{\frac{3n(n+1)}{2}} \qquad (n=0,1,2,3,\ldots) \tag{1}$$

The rigid-body mode corresponds to $n=1$ and has the frequency $\omega_n = \sqrt{3}\omega_c$, as noted in Eq. (5) of Ref. 1. The lower limit of the first flexural mode, corresponding to $n=2$, is $\omega_n = 3\omega_c$. Thus, the ratio ω_n/ω_c is at least equal to 3 for uniform gravity gradient stabilized beams.

This is not to say that the general class of instability studied in Ref. 1 (parametric resonance) cannot occur in the presence of a gravitational potential. Study of the Mathieu-Hill equation indicates small unstable regions near *every* integer and half-integer value of the ratio of natural frequency to the frequency of parametric excitation. Direct application of this observation to Eq. (6) of Ref. 1 shows that the frequencies of parametric excitation are the pitch frequency and twice the pitch frequency. Thus, the centers of the unstable regions are near the half-integer and integer harmonics of the pitch frequency, and not near the orbital frequency as stated in Ref. 1.

References

[1] Kumar, V.K. and Bainum, P.M., "Dynamics of a Flexible Body in Orbit," *Journal of Guidance and Control*, Vol. 3, Jan.-Feb. 1980, pp. 90-92.

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Reply by Authors to R.H. MacNeal

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THE authors would like to acknowledge the interest that Mr. MacNeal has shown in our recent Engineering Note.[1] Mr. MacNeal's comments are apparently based on the following model of a tensioned string in orbit which he claims to be a suitable model as the beam becomes more slender. The string, of length $2l$, is assumed to be rotating at a uniform angular velocity of ω_c (orbital angular velocity). At any section located at a distance ξ from the center of the string, the tension due to the combined centrifugal and gravity-gradient effects is given by

$$T(\xi) = (3/2)\omega_c^2(l^2 - \xi^2) \tag{1}$$

The free transverse vibrations of a string with tension varying along the length of the string may be described by the following partial differential equation[2]:

$$\frac{\partial}{\partial \xi}\left[T(\xi)\frac{\partial y}{\partial \xi}(\xi,t)\right] = \rho\frac{\partial^2 y}{\partial t^2}(\xi,t) \tag{2}$$

where

y = transverse displaement of the string
ρ = mass per unit length of the string
t = time

After substitution of $T(\xi)$, given in Eq. (1), into Eq. (2), the following partial differential equation is obtained:

$$(1-x^2)\frac{\partial^2 y}{\partial x^2} - 2x\frac{\partial y}{\partial x} = \frac{2}{3\omega_c^2}\frac{\partial^2 y}{\partial t^2} \tag{3}$$

where

$$x = \xi/l \qquad -1 \le x \le 1$$

With the customary assumed product solution, $y(x,t) = Z(x)f(t)$, one arrives at the following ordinary differential equations:

$$\ddot{f} + (3/2)c\omega_c^2 f = 0 \tag{4}$$

$$(1-x^2)Z'' - 2xZ' + cZ = 0 \tag{5}$$

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